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Neutrosophic Joint Set

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Report | Exposition | References | Research #22 2022

Abstract

In this book, some notions are introduced about “Neutrosophic Joint Set”. Two chapters are devised as “Initial Notions”, and “Modified Notions”. Two manuscripts are cited as the references of these chapters which are my 81st, and 82nd manuscripts. I’ve used my 81st, and 82nd manuscripts to write this book. In first chapter, there are some points as follow. New setting is introduced to study joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-dominated vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-dominated vertices corresponded to joint-dominating set is called neutrosophic joint-dominating number. Forming sets from joint-dominated vertices to figure out different types of number of vertices in the sets from joint-dominated sets in the terms of minimum number of vertices to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having smallest number of joint-dominated vertices from different types of sets in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there’s at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there’s a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it’s denoted by $\mathcal{J}(NTG)$; for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there’s at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there’s a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called neutrosophic joint-dominating number and it’s denoted by $\mathcal{J}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-dominating number," and "Setting of neutrosophic joint-dominating number," for introduced results and used classes. This approach facilitates identifying sets which form joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of joint-dominated vertices and neutrosophic cardinality of set of joint-dominated vertices corresponded to joint-dominating set have eligibility to define joint-dominating number and neutrosophic joint-dominating number but different types of set of joint-dominated vertices to define joint-dominating sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of joint-dominated vertices in the terms of minimum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic joint-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter. In second chapter, there are some points as follow. New setting is introduced to study joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-resolved vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-resolved vertices corresponded to joint-resolving set is called neutrosophic joint-resolving number. Forming sets from joint-resolved vertices to figure out different types of number of vertices in the sets from joint-resolved sets in the terms of minimum number of vertices to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having smallest number of joint-resolved vertices from different types of sets in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called joint-resolving set where for every two vertices in S, there's a path

in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(NTG)$; for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-resolving number," and "Setting of neutrosophic joint-resolving number," for introduced results and used classes. This approach facilitates identifying sets which form joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of joint-resolved vertices and neutrosophic cardinality of set of joint-resolved vertices corresponded to joint-resolving set have eligibility to define joint-resolving number and neutrosophic joint-resolving number but different types of set of joint-resolved vertices to define joint-resolving sets. Some results get more frameworks and more perspectives about these definitions. The way in that, different types of set of joint-resolved vertices in the terms of minimum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic joint-resolving notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter.
[**Ref1**] Henry Garrett, "*Repetitive Joint-Sets Featuring Multiple Numbers For Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.15113.93283).
[**Ref2**] Henry Garrett, "*Separate Joint-Sets Representing Separate Numbers Where Classes of Neutrosophic Graphs and Applications are Cases of Study*", ResearchGate 2022 (doi: 10.13140/RG.2.2.22666.95686).

Two chapters are devised as "Initial Notions", and "Modified Notions'.

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The words of mind and the minds of words, are too eligible to be in the stage of acknowledgements

Contents

List of Figures

List of Tables

CHAPTER 1

Initial Notions

The following sections are cited as follows, which is my 81st manuscript and I use prefix 81 as number before any labelling for items.

[**Ref1**] Henry Garrett, "*Repetitive Joint-Sets Featuring Multiple Numbers For Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.15113.93283).

Repetitive Joint-Sets Featuring Multiple Numbers For Neutrosophic Graphs

1.1 Abstract

New setting is introduced to study joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-dominated vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-dominated vertices corresponded to joint-dominating set is called neutrosophic joint-dominating number. Forming sets from joint-dominated vertices to figure out different types of number of vertices in the sets from joint-dominated sets in the terms of minimum number of vertices to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having smallest number of joint-dominated vertices from different types of sets in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(NTG)$; for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices

[a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called neutrosophic joint-dominating number and it's denoted by $\mathcal{J}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-dominating number," and "Setting of neutrosophic joint-dominating number," for introduced results and used classes. This approach facilitates identifying sets which form joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of joint-dominated vertices and neutrosophic cardinality of set of joint-dominated vertices corresponded to joint-dominating set have eligibility to define joint-dominating number and neutrosophic joint-dominating number but different types of set of joint-dominated vertices to define joint-dominating sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of joint-dominated vertices in the terms of minimum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic joint-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Joint-Dominating Number, Neutrosophic Joint-Dominating

Number, Classes of Neutrosophic Graphs
AMS Subject Classification: 05C17, 05C22, 05E45

1.2 Background

Fuzzy set in **Ref.** [**Ref20**] by Zadeh (1965), intuitionistic fuzzy sets in **Ref.** [**Ref3**] by Atanassov (1986), a first step to a theory of the intuitionistic fuzzy graphs in **Ref.** [**Ref17**] by Shannon and Atanassov (1994), a unifying field in logics neutrosophy: neutrosophic probability, set and logic, rehoboth in **Ref.** [**Ref18**] by Smarandache (1998), single-valued neutrosophic sets in **Ref.** [**Ref19**] by Wang et al. (2010), single-valued neutrosophic graphs in **Ref.** [**Ref6**] by Broumi et al. (2016), operations on single-valued neutrosophic graphs in **Ref.** [**Ref1**] by Akram and Shahzadi (2017), neutrosophic soft graphs in **Ref.** [**Ref16**] by Shah and Hussain (2016), bounds on the average and minimum attendance in preference-based activity scheduling in **Ref.** [**Ref2**] by

Aronshtam and Ilani (2022), investigating the recoverable robust single machine scheduling problem under interval uncertainty in **Ref.** [**Ref5**] by Bold and Goerigk (2022), some variants of perfect graphs related to the matching number, the vertex cover and the weakly connected domination number in **Ref.** [**Ref4**] by S. Bermudo et al. (2021), bounds for the connected domination number of maximal outerplanar graphs in **Ref.** [**Ref7**] by S.L. Chen et al. (2022), bounds on the connected domination number of a graph in **Ref.** [**Ref8**] by W.J. Desormeaux et al. (2013), a conjecture on the lower bound of the signed edge domination number of 2-connected graphs in **Ref.** [**Ref9**] by X. Feng, and J. Ge (2021), complexity of total outer-connected domination problem in graphs in **Ref.** [**Ref14**] by B.S. Panda, and A. Pandey (2016), weakly connected Roman domination in graphs in **Ref.** [**Ref15**] by J. Raczek, and J. Cyman (2019), connected domination in maximal outerplanar graphs in **Ref.** [**Ref21**] by W. Zhuang (2020), on graphs for which the connected domination number is at most the total domination number in **Ref.** [**Ref22**] by W. Zhuang (2012), dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [**Ref9**] by Henry Garrett (2022), three types of neutrosophic alliances based on connectedness and (strong) edges in **Ref.** [**Ref11**] by Henry Garrett (2022), properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [**Ref10**] by Henry Garrett (2022), are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [**Ref8**] by Henry Garrett (2022).
In this section, I use two sections to illustrate a perspective about the background of this study.

1.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.3.1. *Is it possible to use mixed versions of ideas concerning "joint-dominating number", "neutrosophic joint-dominating number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having smallest number of joint-dominated vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all joint-dominated vertices in the way that, some types of numbers, joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce

basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", minimum number of joint-dominated vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, in section "Setting of joint-dominating number," as individuals. In section "Setting of joint-dominating number," joint-dominating number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-dominating number," and "Setting of neutrosophic joint-dominating number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The

added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sum_{i=1}^{3}\sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E$, $i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E$, $i = 0, 1, \cdots, n-1$, $x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 1.4.6. Using notations which is mixed with literatures, are reviewed.

1.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$, $\mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

1.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

Definition 1.4.7. (joint-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called **joint-dominating number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called **neutrosophic joint-dominating number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

81prp9

Proposition 1.4.8. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it joint-dominates.*

Proposition 1.4.9. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S is corresponded to joint-dominating number. Then $V \setminus D$ is S-like.*

Proposition 1.4.10. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{n' \mid n'n \in E\} \cap S = \{s\}$.*

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.4.11. In Figure (1.1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) one vertex dominates all other vertices thus by there's only one member for S and Proposition (1.4.8), this vertex joint-dominates other vertices;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(NTG) = 1$;

(iv) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

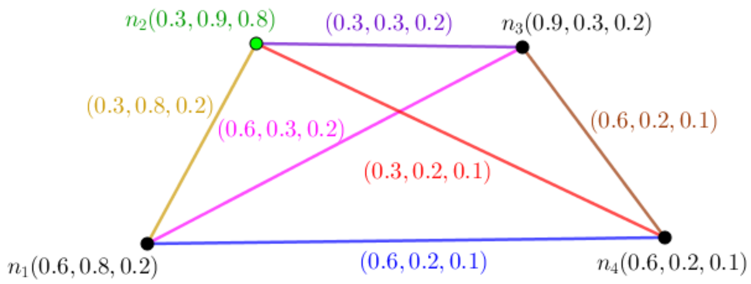

Figure 1.1: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG1

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(NTG) = 0.9$.

1.5 Setting of joint-dominating number

In this section, I provide some results in the setting of joint-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 1.5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in

S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$. Thus

$$\mathcal{J}(CMT_\sigma) = 1.$$

■

Proposition 1.5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-dominating number is equal to dominating number.*

Proof. S has one member thus by Proposition (1.4.8), the result holds. ■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.3. In Figure (1.2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) one vertex dominates all other vertices thus by there's only one member for S and Proposition (1.4.8), this vertex joint-dominates other vertices;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$;

(iv) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like

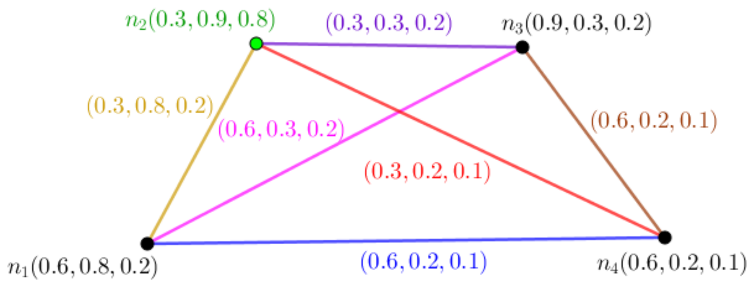

Figure 1.2: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number.

81NTG2

$\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 0.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.5.4. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, x and y, there's one path from x to y. Let S be an intended set which is joint-dominating set corresponded to joint-dominating number. Despite two leaves $x'_{\mathcal{O}(PTH)}$ and $x'_{\mathcal{O}(PTH)-1}$, all neutrosophic vertices belong to S corresponded to joint-dominating number. Leaves could be joint-dominated by their unique neighbors $x'_{\mathcal{O}(PTH)-2}$ and $x'_{\mathcal{O}(PTH)-3}$ as if despite the leaves $x'_{\mathcal{O}(PTH)}$ and $x'_{\mathcal{O}(PTH)-1}$, so as maximal set S is constructed. Thus $S = \{x'_1, x'_2, \cdots, x'_{\mathcal{O}(PTH)-2}\}$ is the set S is a set of vertices from path-neutrosophic graph $PTH : (V, E, \sigma, \mu)$ with new arrangements of vertices in which there are all neutrosophic vertices which are either neighbors or not. In these new arrangements, the notation of vertices from x_i is changed to x'_i. Leaves doesn't necessarily belong to S. Leaves are either belongs to S or doesn't belong to S as if Leaves doesn't belong to S. corresponded to joint-dominating number. Adding all neutrosophic leaves contradicts with maximality of S corresponded to joint-dominating number and maximum cardinality of S corresponded to joint-dominating number. It implies this construction is optimal. Thus, let

$$S = \{x'_1, x'_2, \cdots, x'_{\mathcal{O}(PTH)-3}, x'_{\mathcal{O}(PTH)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates

n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2$. Thus

$$\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2.$$

■

Proposition 1.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are four joint-dominating sets.*

Proposition 1.5.6. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there's one joint-dominating set corresponded to joint-dominating number.*

Example 1.5.7. There are two sections for clarifications.

(a) In Figure (1.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, s and s', there's only one path between them;
- (ii) one vertex only dominates either two vertices or one vertex if it isn't a leaf, then it only dominates its two neighbors and if it's a leaf, then it only dominates its one neighbor thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of path;
- (iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_2, n_3, n_4\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2 = 3$;
- (iv) there are four joint-dominating sets $\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}$ and $\{n_1, n_2, n_3, n_4, n_5\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;
- (v) there are four joint-dominating sets $\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}$ and $\{n_1, n_2, n_3, n_4, n_5\}$ as if there is one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;
- (vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_2, n_3, n_4\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices

[a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_5)) = 3.7$.

(b) In Figure (1.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's only one path between them;

(ii) one vertex only dominates either two vertices or one vertex if it isn't a leaf, then it only dominates its two neighbors and if it's a leaf, then it only dominates its one neighbor thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of path;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_2, n_3, n_4, n_5\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2 = 4$;

(iv) there are four joint-dominating sets $\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5, n_6\}$ and $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5, n_6\}$ and $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ as if there is one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_2, n_3, n_4, n_5\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_6)) = 6$.

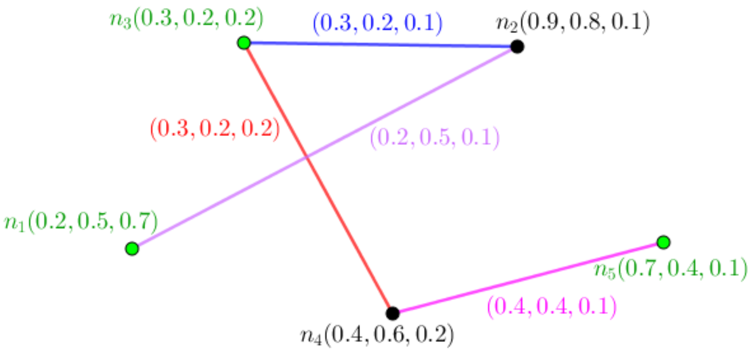

Figure 1.3: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG3

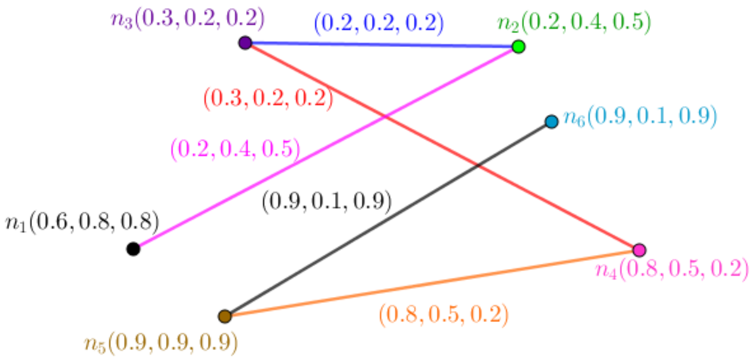

Figure 1.4: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG4

Proposition 1.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is joint-dominating set related to joint-dominating number where two neutrosophic vertices outside are "consecutive". Since it's possible to have a path amid every two of vertices in S and two vertices outside could be joint-dominated by their neighbors in S. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-dominating set for S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in

$$V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}),$$

there's only one neutrosophic vertex s in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

is called joint-dominating set where for every two vertices in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\},$$

there's only one path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

Thus

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

■

Proposition 1.5.9. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $3 \times \mathcal{O}(CYC) + 1$ joint-dominating sets.*

Proposition 1.5.10. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.11. There are two sections for clarifications.

(a) In Figure (1.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

   (i) For given two neutrosophic vertices, s and s', there are only two paths between them;

   (ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}.$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 4$;

(iv) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if it's possible to have six of them

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if there is six joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4, n_5, n_6, n_1\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_2) + \sigma(n_3)) = 4.1$.

(b) In Figure (1.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there are only two paths between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 3$;

(iv) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have five of them

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

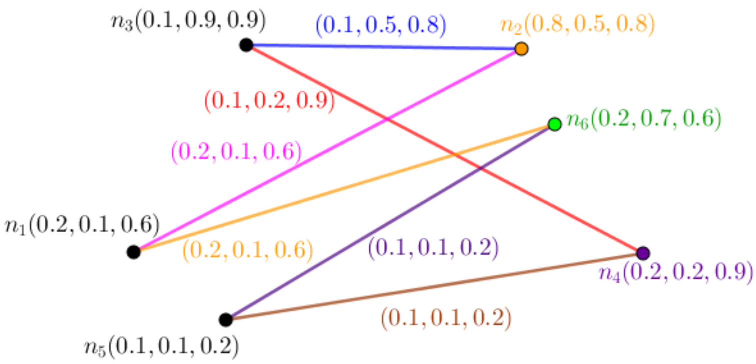

Figure 1.5: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number.

81NTG5

(v) there are sixteen joint-dominating sets

$$\begin{aligned}&\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},\\ &\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},\\ &\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},\\ &\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},\\ &\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},\\ &\{n_1, n_2, n_3, n_4, n_5\},\end{aligned}$$

as if there is five joint-dominating sets

$$\begin{aligned}&\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},\\ &\{n_5, n_1, n_2\},\end{aligned}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_5, n_1, n_2\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_3) + \sigma(n_4)) = 4.2.$

Proposition 1.5.12. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{J}(STR_{1,\sigma_2}) = 1.$$

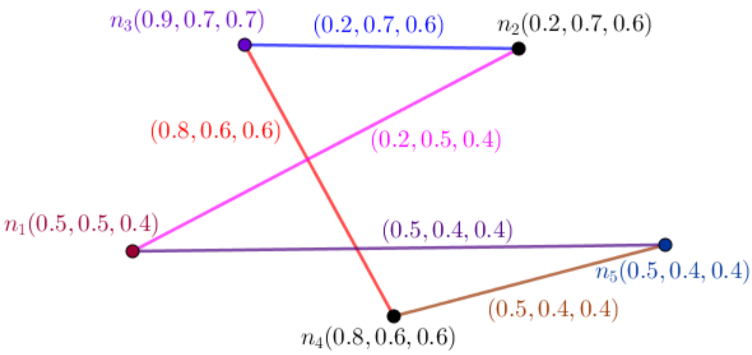

Figure 1.6: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG6

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. Every given vertex n implies $cn \in E$, then c joint-dominates n. $S = \{c\}$ is a joint-dominating set related joint-dominating number. Since, let

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex c in

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

such that c joint-dominates n, then the set of neutrosophic vertices,

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(STR_{1,\sigma_2}) = 1.$$

Thus

$$\mathcal{J}(STR_{1,\sigma_2}) = 1.$$

∎

Proposition 1.5.13. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $2^{\mathcal{O}(STR_{1,\sigma_2})-1} - 1$ joint-dominating sets.*

Proposition 1.5.14. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there's one joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it.

To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.15. There is one section for clarifications. In Figure (1.7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there are only one path, precisely one edge between them and there's no path despite them;

(ii) one vertex only dominates one vertex, then it only dominates its neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of star, by Proposition (1.4.8) and S has one member;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1\}.$$

For given vertex n if $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(STR_{1,\sigma_2}) = 1$;

(iv) there are fifteen joint-dominating sets

$$\begin{aligned}
&\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},\\
&\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\
&\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},\\
&\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},\\
&\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},
\end{aligned}$$

as if it's possible to have one of them

$$\{n_1\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are fifteen joint-dominating sets

$$\begin{aligned}
&\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},\\
&\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\
&\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},\\
&\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},\\
&\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},
\end{aligned}$$

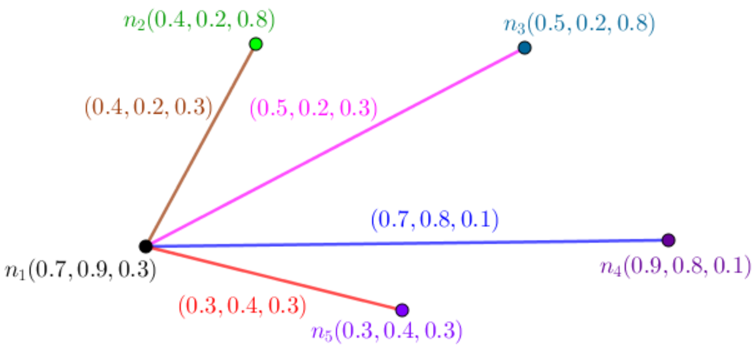

Figure 1.7: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG7

as if there's one joint-dominating set

$$\{n_1\}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_1\}$. Every given vertex n implies $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(STR_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma(n_1) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3}(\sigma(n_2)+\sigma(n_3)+\sigma(n_4)+\sigma(n_5)) = 1.9$.

Proposition 1.5.16. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-dominated by any given vertex. Thus minimum cardinality implies including one vertex from each part. Let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

be a joint-dominating set related to the joint-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex u in $V \setminus S$, there's a neutrosophic vertex n in

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

such that n joint-dominates u, then the set of neutrosophic vertices,

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by the property of completeness amid two vertices from different parts. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = 2.$$

Thus

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = 2.$$

■

Proposition 1.5.17. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then there are $|V_1| \times |V_2|$ joint-dominating sets corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.18. There is one section for clarifications. In Figure (1.8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},$$
$$\{n_4, n_3\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates

n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = 2$;

(*iv*) there are ten joint-dominating sets

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\ &\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\},\\ &\{n_2, n_4, n_3\}, \{n_3, n_4\}, \{n_3, n_4, n_2\},\\ &\{n_1, n_2, n_3, n_4\},\end{aligned}$$

as if it's possible to have four of them

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},\\ &\{n_4, n_3\},\end{aligned}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are ten joint-dominating sets

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\ &\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\},\\ &\{n_2, n_4, n_3\}, \{n_3, n_4\}, \{n_3, n_4, n_2\},\\ &\{n_1, n_2, n_3, n_4\},\end{aligned}$$

as if there's four joint-dominating sets

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},\\ &\{n_4, n_3\},\end{aligned}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(*vi*) there are only four joint-dominating sets corresponded to joint-dominating number are

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},\\ &\{n_4, n_3\}.\end{aligned}$$

For given vertex n, if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \sum_{i=1}^{3}(\sigma(n_4) + \sigma(n_2)) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_3)) = 2.4$.

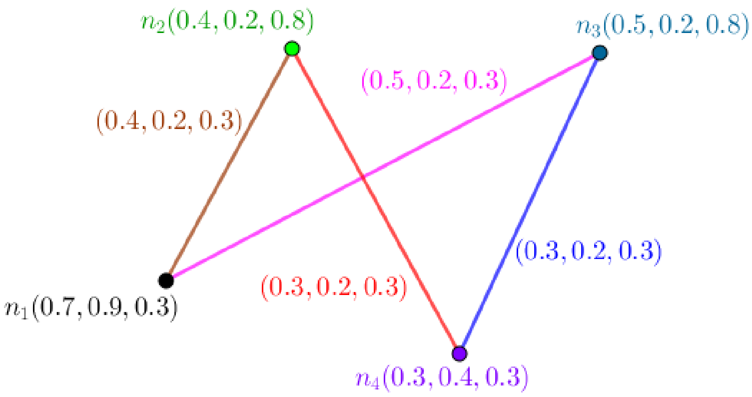

Figure 1.8: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG8

Proposition 1.5.19. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-dominated by any given vertex. Thus minimum cardinality implies including two vertices from two different parts. Let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a joint-dominating set related to the joint-dominating number. This construction gives the proof. Since let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's a neutrosophic vertex s in

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2},$$

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

is called joint-dominating set. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

Thus

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.$$

■

Proposition 1.5.20. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then there are $|V_1| \times |V_2| \times \ldots \times |V_{t-1}| \times |V_t|$ joint-dominating sets corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.21. There is one section for clarifications. In Figure (1.9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) there are twenty-one joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3\}, \{n_1, n_3, n_4\},$$
$$\{n_1, n_3, n_5\}, \{n_2, n_4\}, \{n_2, n_4, n_3\},$$

$$\{n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_5\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have six of them

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are twenty-one joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3\}, \{n_1, n_3, n_4\},$$
$$\{n_1, n_3, n_5\}, \{n_2, n_4\}, \{n_2, n_4, n_3\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_5\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if there's six joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there are only six joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

If for given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \sum_{i=1}^{3}(\sigma(n_4) + \sigma(n_2)) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_3) + \sigma(n_5)) = 2.4.$

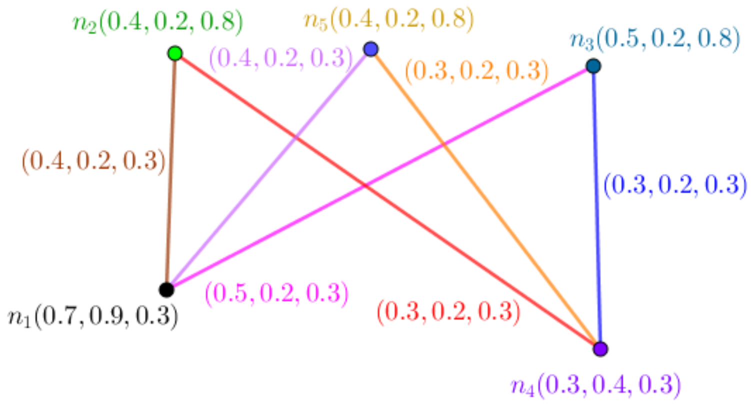

Figure 1.9: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG9

Proposition 1.5.22. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{J}(WHL_{1,\sigma_2}) = 1.$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. $S = \{c\}$ is a joint-dominating set related joint-dominating number. Since, let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's a neutrosophic vertex n in

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

is called joint-dominating set. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(WHL_{1,\sigma_2}) = 1.$$

Thus

$$\mathcal{J}(WHL_{1,\sigma_2}) = 1.$$

■

Proposition 1.5.23. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $2^{\mathcal{O}(WHL_{1,\sigma_2})-1} - 1$ joint-dominating sets.*

Proposition 1.5.24. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there's one joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.25. There is one section for clarifications. In Figure (1.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there are only one edge between them;

(ii) one vertex dominates some vertices, then it only dominates its neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of star, by Proposition (1.4.8) and S has one member;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1\}.$$

Every given vertex n implies $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(WHL_{1,\sigma_2}) = 1$;

(iv) there are fifteen joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},$$

as if it's possible to have one of them

$$\{n_1\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are fifteen joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},$$

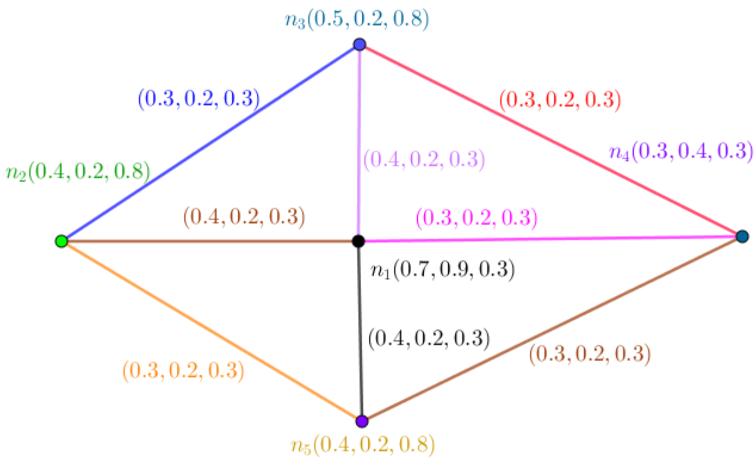

Figure 1.10: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG10

as if there's one joint-dominating set

$$\{n_1\}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_1\}$. Every given vertex n implies $n_1 n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(WHL_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma(n_1) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} (\sigma(n_2)+\sigma(n_3)+\sigma(n_4)+\sigma(n_5)) = 1.9$.

1.6 Setting of neutrosophic joint-dominating number

In this section, I provide some results in the setting of neutrosophic joint-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 1.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Thus

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 1.6.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-dominating number is equal to dominating number.*

Proof. S has one member thus by Proposition (1.4.8), the result holds. ■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.3. In Figure (1.11), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) one vertex dominates all other vertices thus by there's only one member for S and Proposition (1.4.8), this vertex joint-dominates other vertices;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$.

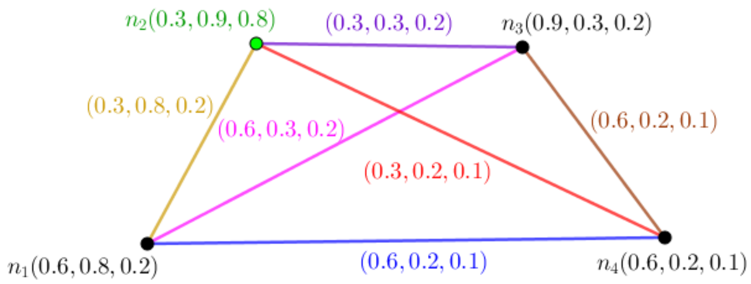

Figure 1.11: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG11

is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$;

(iv) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 0.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.6.4. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y))_{x \text{ and } y \text{ are leaves}}.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices,

x and y, there's one path from x to y. Let S be an intended set which is joint-dominating set corresponded to joint-dominating number. Despite two leaves $x'_{\mathcal{O}(PTH)}$ and $x'_{\mathcal{O}(PTH)-1}$, all neutrosophic vertices belong to S corresponded to joint-dominating number. Leaves could be joint-dominated by their unique neighbors $x'_{\mathcal{O}(PTH)-2}$ and $x'_{\mathcal{O}(PTH)-3}$ as if despite the leaves $x'_{\mathcal{O}(PTH)}$ and $x'_{\mathcal{O}(PTH)-1}$, so as maximal set S is constructed. Thus $S = \{x'_1, x'_2, \cdots, x'_{\mathcal{O}(PTH)-2}\}$ is the set S is a set of vertices from path-neutrosophic graph $PTH : (V, E, \sigma, \mu)$ with new arrangements of vertices in which there are all neutrosophic vertices which are either neighbors or not. In these new arrangements, the notation of vertices from x_i is changed to x'_i. Leaves doesn't necessarily belong to S. Leaves are either belongs to S or doesn't belong to S as if Leaves doesn't belong to S. corresponded to joint-dominating number. Adding all neutrosophic leaves contradicts with maximality of S corresponded to joint-dominating number and maximum cardinality of S corresponded to joint-dominating number. It implies this construction is optimal. Thus, let

$$S = \{x'_1, x'_2, \cdots, x'_{\mathcal{O}(PTH)-3}, x'_{\mathcal{O}(PTH)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))_{x \text{ and } y \text{ are leaves}}.$$

Thus

$$\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))_{x \text{ and } y \text{ are leaves}}.$$

■

Proposition 1.6.5. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are four joint-dominating sets.*

Proposition 1.6.6. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there's one joint-dominating set corresponded to joint-dominating number.*

Example 1.6.7. There are two sections for clarifications.

(a) In Figure (1.12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given two neutrosophic vertices, s and s', there's only one path between them;

(ii) one vertex only dominates either two vertices or one vertex if it isn't a leaf, then it only dominates its two neighbors and if it's a leaf, then it only dominates its one neighbor thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of path;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_2, n_3, n_4\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2 = 3$;

(iv) there are four joint-dominating sets $\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}$ and $\{n_1, n_2, n_3, n_4, n_5\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}$ and $\{n_1, n_2, n_3, n_4, n_5\}$ as if there is one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_2, n_3, n_4\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_5)) = 3.7$.

(b) In Figure (1.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's only one path between them;

(ii) one vertex only dominates either two vertices or one vertex if it isn't a leaf, then it only dominates its two neighbors and if it's a leaf, then it only dominates its one neighbor thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of path;

(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_2, n_3, n_4, n_5\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.].

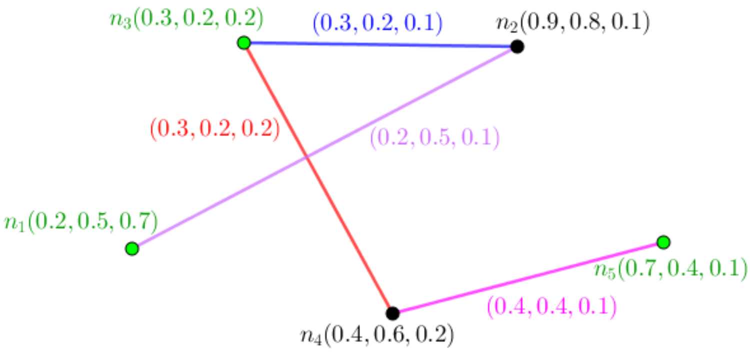

Figure 1.12: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG12

If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(PTH) = \mathcal{O}(PTH) - 2 = 4$;

(iv) there are four joint-dominating sets $\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5, n_6\}$ and $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5, n_6\}$ and $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ as if there is one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_2, n_3, n_4, n_5\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(PTH) = \mathcal{O}_n(PTH) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_6)) = 6$.

Proposition 1.6.8. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y)).$$

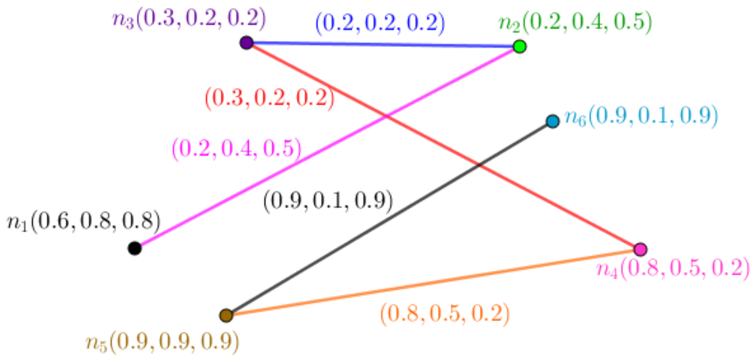

Figure 1.13: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG13

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is joint-dominating set related to joint-dominating number where two neutrosophic vertices outside are "consecutive". Since it's possible to have a path amid every two of vertices in S and two vertices outside could be joint-dominated by their neighbors in S. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-dominating set for S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in

$$V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}),$$

there's only one neutrosophic vertex s in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

is called joint-dominating set where for every two vertices in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\},$$

there's only one path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

■

Proposition 1.6.9. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $3 \times \mathcal{O}(CYC) + 1$ joint-dominating sets.*

Proposition 1.6.10. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.11. There are two sections for clarifications.

(a) In Figure (1.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, s and s', there are only two paths between them;
- (ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;
- (iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}.$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 4$;

(iv) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if it's possible to have six of them

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if there is six joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4, n_5, n_6, n_1\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_2) + \sigma(n_3)) = 4.1$.

(b) In Figure (1.15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given two neutrosophic vertices, s and s', there are only two paths between them;

(*ii*) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(*iii*) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 3$;

(*iv*) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have five of them

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if there is five joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

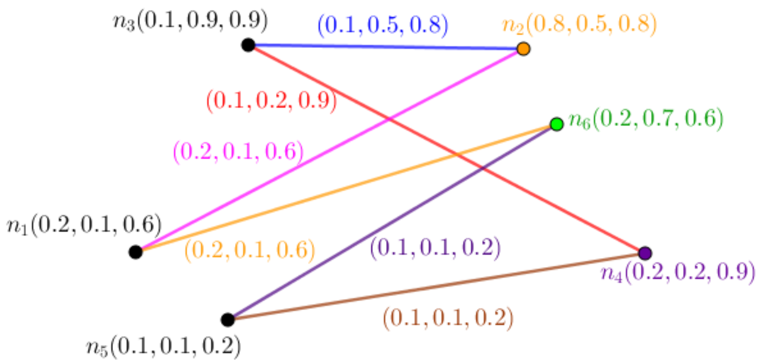

Figure 1.14: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG14

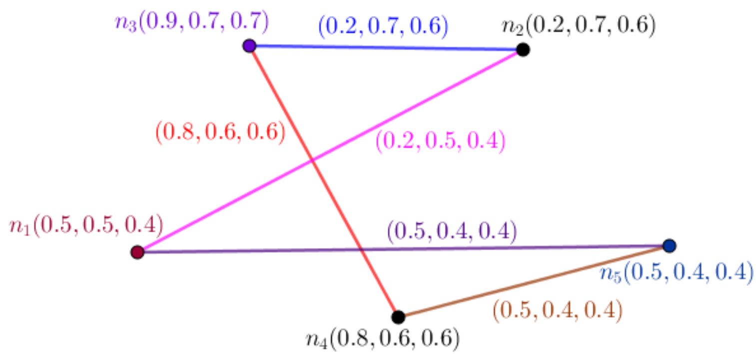

Figure 1.15: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG15

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_5, n_1, n_2\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_3) + \sigma(n_4)) = 4.2$.

Proposition 1.6.12. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. Every given vertex n implies $cn \in E$, then c joint-dominates n. $S = \{c\}$ is a joint-dominating set related joint-dominating number. Since, let

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex c in

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

such that c joint-dominates n, then the set of neutrosophic vertices,

$$S = \{c\} = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

■

Proposition 1.6.13. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $2^{\mathcal{O}(STR_{1,\sigma_2})-1} - 1$ joint-dominating sets.*

Proposition 1.6.14. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there's one joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.15. There is one section for clarifications. In Figure (1.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there are only one path, precisely one edge between them and there's no path despite them;

(ii) one vertex only dominates one vertex, then it only dominates its neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of star, by Proposition (1.4.8) and S has one member;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1\}.$$

For given vertex n if $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(STR_{1,\sigma_2}) = 1$;

(iv) there are fifteen joint-dominating sets

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},\\&\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\&\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},\\&\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},\\&\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},\end{aligned}$$

as if it's possible to have one of them

$$\{n_1\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are fifteen joint-dominating sets

$$\begin{aligned}&\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},\\&\{n_1, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\&\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},\\&\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},\\&\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},\end{aligned}$$

as if there's one joint-dominating set

$$\{n_1\}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_1\}$. Every given vertex n implies $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's

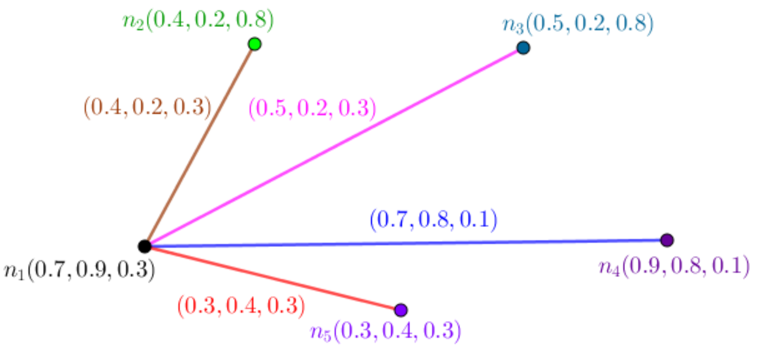

Figure 1.16: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG16

a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(STR_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma(n_1) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3}(\sigma(n_2)+\sigma(n_3)+\sigma(n_4)+\sigma(n_5)) = 1.9.$

Proposition 1.6.16. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \min_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-dominated by any given vertex. Thus minimum cardinality implies including one vertex from each part. Let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

be a joint-dominating set related to the joint-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex u in $V \setminus S$, there's a neutrosophic vertex n in

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

such that n joint-dominates u, then the set of neutrosophic vertices,

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\} = \{u, v\}_{u \in V_1, v \in V_2}$$

is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by the property of completeness amid two vertices from different parts. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \min_{x\in V_1, y\in V_2} \sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)).$$

Thus

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \min_{x\in V_1, y\in V_2} \sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)).$$

■

Proposition 1.6.17. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then there are $|V_1| \times |V_2|$ joint-dominating sets corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.18. There is one section for clarifications. In Figure (1.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},$$
$$\{n_4, n_3\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = 2$;

(iv) there are ten joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$

$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_4, n_3\}, \{n_3, n_4\}, \{n_3, n_4, n_2\},$$
$$\{n_1, n_2, n_3, n_4\},$$

as if it's possible to have four of them

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},$$
$$\{n_4, n_3\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are ten joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_4, n_3\}, \{n_3, n_4\}, \{n_3, n_4, n_2\},$$
$$\{n_1, n_2, n_3, n_4\},$$

as if there's four joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},$$
$$\{n_4, n_3\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there are only four joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_4, n_2\},$$
$$\{n_4, n_3\}.$$

For given vertex n, if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \sum_{i=1}^{3}(\sigma(n_4) + \sigma(n_2)) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_3)) = 2.4$.

Proposition 1.6.19. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y)).$$

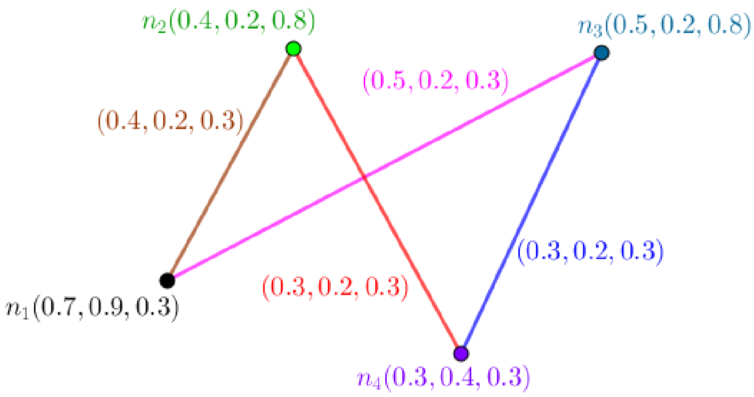

Figure 1.17: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG17

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-dominated by any given vertex. Thus minimum cardinality implies including two vertices from two different parts. Let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a joint-dominating set related to the joint-dominating number. This construction gives the proof. Since let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's a neutrosophic vertex s in

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2},$$

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

is called joint-dominating set. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus
$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_{x\in V_i, y\in V_j, i\neq j} \sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)).$$

■

Proposition 1.6.20. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then there are $|V_1| \times |V_2| \times \ldots \times |V_{t-1}| \times |V_t|$ joint-dominating sets corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.21. There is one section for clarifications. In Figure (1.18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) there are twenty-one joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3\}, \{n_1, n_3, n_4\},$$
$$\{n_1, n_3, n_5\}, \{n_2, n_4\}, \{n_2, n_4, n_3\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_5\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have six of them

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are twenty-one joint-dominating sets

$$\begin{aligned}
&\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\
&\{n_1, n_2, n_5\}, \{n_1, n_3\}, \{n_1, n_3, n_4\},\\
&\{n_1, n_3, n_5\}, \{n_2, n_4\}, \{n_2, n_4, n_3\},\\
&\{n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_5\},\\
&\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_1, n_2, n_3, n_4\},\\
&\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},\\
&\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},
\end{aligned}$$

as if there's six joint-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there are only six joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

If for given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \sum_{i=1}^{3}(\sigma(n_4) + \sigma(n_2)) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma(n_1) + \sigma(n_3) + \sigma(n_5)) = 2.4$.

Proposition 1.6.22. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

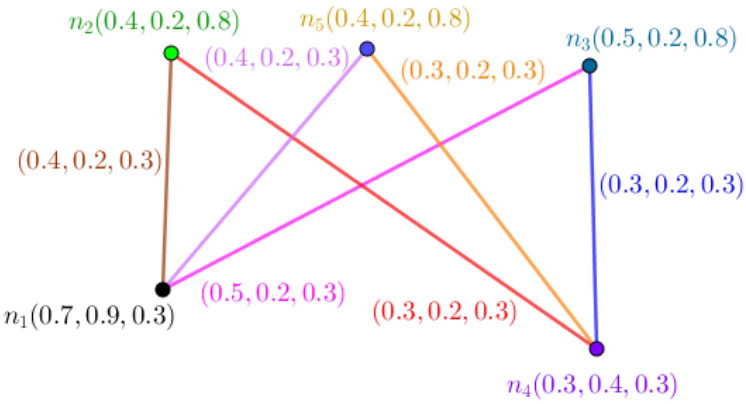

Figure 1.18: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG18

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. $S = \{c\}$ is a joint-dominating set related joint-dominating number. Since, let

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's a neutrosophic vertex n in

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = V \setminus \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} = \{c\}$$

is called joint-dominating set. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma_i(c).$$

∎

Proposition 1.6.23. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $2^{\mathcal{O}(WHL_{1,\sigma_2})-1} - 1$ joint-dominating sets.*

Proposition 1.6.24. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there's one joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.25. There is one section for clarifications. In Figure (1.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there are only one edge between them;

(ii) one vertex dominates some vertices, then it only dominates its neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of star, by Proposition (1.4.8) and S has one member;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1\}.$$

Every given vertex n implies $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(WHL_{1,\sigma_2}) = 1$;

(iv) there are fifteen joint-dominating sets

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_4\},$$
$$\{n_1,n_5\},\{n_1,n_2,n_3\},\{n_1,n_2,n_4\},$$
$$\{n_1,n_2,n_5\},\{n_1,n_3,n_4\},\{n_1,n_3,n_5\},$$
$$\{n_1,n_4,n_5\},\{n_1,n_2,n_3,n_4\},\{n_1,n_2,n_3,n_5\},$$
$$\{n_1,n_3,n_4,n_5\},\{n_1,n_2,n_3,n_4,n_5\},\{n_1\},$$

as if it's possible to have one of them

$$\{n_1\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are fifteen joint-dominating sets

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_4\},$$
$$\{n_1,n_5\},\{n_1,n_2,n_3\},\{n_1,n_2,n_4\},$$

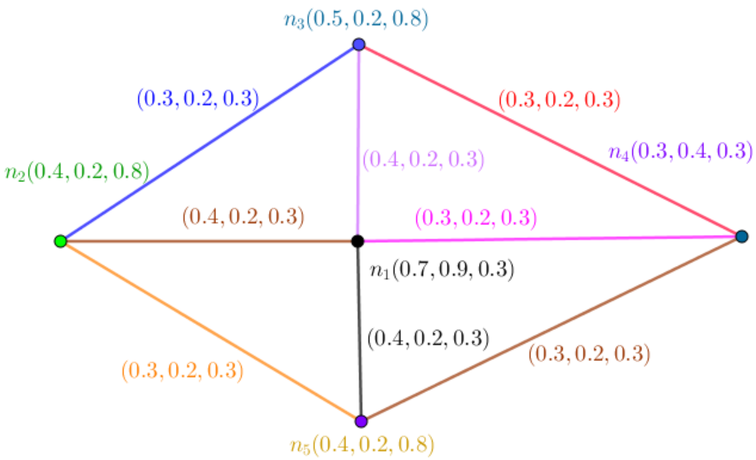

Figure 1.19: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG19

$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_1\},$$

as if there's one joint-dominating set

$$\{n_1\}$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_1\}$. Every given vertex n implies $n_1n \in E$, then n_1 joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex n_1 in S such that n_1 joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them, by Proposition (1.4.8) and S has one member. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(WHL_{1,\sigma_2}) = \sum_{i=1}^{3} \sigma(n_1) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3}(\sigma(n_2)+\sigma(n_3)+\sigma(n_4)+\sigma(n_5)) = 1.9$.

1.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common

neutrosophic vertex set.
Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1.1), clarifies about the assigned numbers to these situations.

Table 1.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 81tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

1.8 Case 1: Complete-t-partite Model alongside its joint-dominating number and its neutrosophic joint-dominating number

Step 4. (Solution) The neutrosophic graph alongside its joint-dominating number and its neutrosophic joint-dominating number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its joint-dominating number and its neutrosophic joint-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (1.20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its joint-dominating number and its neutrosophic

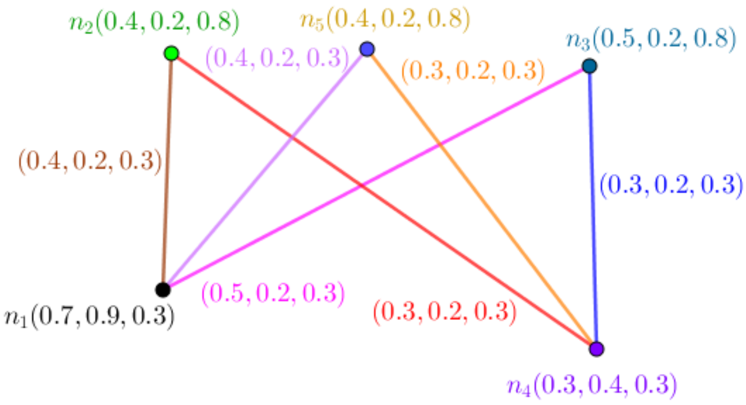

Figure 1.20: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number 81NTG20

joint-dominating number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (1.20). In Figure (1.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is as same as vertex dominates vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_5\},$$
$$\{n_4, n_2\}, \{n_4, n_3\}, \{n_4, n_5\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) there are twenty-one joint-dominating sets

$$\{n_1,n_2\},\{n_1,n_2,n_3\},\{n_1,n_2,n_4\},$$
$$\{n_1,n_2,n_5\},\{n_1,n_3\},\{n_1,n_3,n_4\},$$
$$\{n_1,n_3,n_5\},\{n_2,n_4\},\{n_2,n_4,n_3\},$$
$$\{n_2,n_4,n_5\},\{n_3,n_4\},\{n_3,n_4,n_5\},$$
$$\{n_5,n_4\},\{n_5,n_4,n_1\},\{n_1,n_2,n_3,n_4\},$$
$$\{n_1,n_2,n_3,n_5\},\{n_1,n_2,n_4,n_5\},\{n_1,n_3,n_4,n_5\},$$
$$\{n_2,n_3,n_4,n_5\},\{n_1,n_2,n_3,n_4\},\{n_1,n_2,n_3,n_4,n_5\},$$

as if it's possible to have six of them

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_5\},$$
$$\{n_4,n_2\},\{n_4,n_3\},\{n_4,n_5\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are twenty-one joint-dominating sets

$$\{n_1,n_2\},\{n_1,n_2,n_3\},\{n_1,n_2,n_4\},$$
$$\{n_1,n_2,n_5\},\{n_1,n_3\},\{n_1,n_3,n_4\},$$
$$\{n_1,n_3,n_5\},\{n_2,n_4\},\{n_2,n_4,n_3\},$$
$$\{n_2,n_4,n_5\},\{n_3,n_4\},\{n_3,n_4,n_5\},$$
$$\{n_5,n_4\},\{n_5,n_4,n_1\},\{n_1,n_2,n_3,n_4\},$$
$$\{n_1,n_2,n_3,n_5\},\{n_1,n_2,n_4,n_5\},\{n_1,n_3,n_4,n_5\},$$
$$\{n_2,n_3,n_4,n_5\},\{n_1,n_2,n_3,n_4\},\{n_1,n_2,n_3,n_4,n_5\},$$

as if there's six joint-dominating sets

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_5\},$$
$$\{n_4,n_2\},\{n_4,n_3\},\{n_4,n_5\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there are only six joint-dominating sets corresponded to joint-dominating number are

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_5\},$$
$$\{n_4,n_2\},\{n_4,n_3\},\{n_4,n_5\},$$

If for given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path, precisely an edge, in S amid them, by S has two members from different parts implies one edge amid them. The minimum

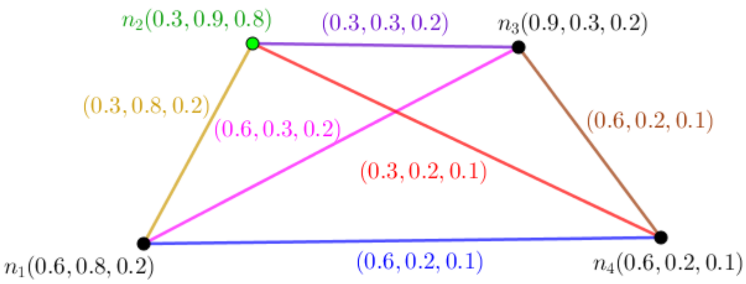

Figure 1.21: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number 81NTG21

neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \sum_{i=1}^{3}(\sigma(n_4)+\sigma(n_2)) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma(n_1)+\sigma(n_3)+\sigma(n_5)) = 2.4.$

1.9 Case 2: Complete Model alongside its Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number

Step 4. (Solution) The neutrosophic graph alongside its joint-dominating number and its neutrosophic joint-dominating number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its joint-dominating number and its neutrosophic joint-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (1.21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its joint-dominating number and its neutrosophic joint-dominating number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider

Figure (1.21). There is one section for clarifications.

(*i*) For given two neutrosophic vertices, s and s', there's an edge between them;

(*ii*) one vertex dominates all other vertices thus by there's only one member for S and Proposition (1.4.8), this vertex joint-dominates other vertices;

(*iii*) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$;

(*iv*) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(*vi*) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (1.4.8), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (1.4.8). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 0.9$.

1.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its joint-dominating number and its neutrosophic joint-dominating number are defined in neutrosophic graphs. Thus,

Question 1.10.1. *Is it possible to use other types of its joint-dominating number and its neutrosophic joint-dominating number?*

Question 1.10.2. *Are existed some connections amid different types of its joint-dominating number and its neutrosophic joint-dominating number in neutrosophic graphs?*

Question 1.10.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 1.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 1.10.5. *Which parameters are related to this parameter?*

Problem 1.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 1.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

1.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses two definitions concerning joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-dominated vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-dominated vertices corresponded to joint-dominating set is called neutrosophic joint-dominating number. The connections of vertices which aren't clarified by minimum number of edges amid them differ them from each other and put them in different categories to

Table 1.2: A Brief Overview about Advantages and Limitations of this Study 81tbl2

Advantages	Limitations
1. Joint-Dominating Number of Model	1. Connections amid Classes
2. Neutrosophic joint-dominating Number of Model	
3. Minimal Joint-Dominating Sets	2. Study on Families
4. Joint-Dominated Vertices amid all Vertices	
5. Acting on All Vertices	3. Same Models in Family

represent a number which is called joint-dominating number and neutrosophic joint-dominating number arising from joint-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (1.2), some limitations and advantages of this study are pointed out.

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CHAPTER 2

Modified Notions

The following sections are cited as follows, which is my 82nd manuscript and I use prefix 82 as number before any labelling for items.

[**Ref2**] Henry Garrett, "*Separate Joint-Sets Representing Separate Numbers Where Classes of Neutrosophic Graphs and Applications are Cases of Study*", ResearchGate 2022 (doi: 10.13140/RG.2.2.22666.95686).

Separate Joint-Sets Representing Separate Numbers Where Classes of Neutrosophic Graphs and Applications are Cases of Study

2.1 Abstract

New setting is introduced to study joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-resolved vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-resolved vertices corresponded to joint-resolving set is called neutrosophic joint-resolving number. Forming sets from joint-resolved vertices to figure out different types of number of vertices in the sets from joint-resolved sets in the terms of minimum number of vertices to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having smallest number of joint-resolved vertices from different types of sets in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called joint-resolving set where for every two vertices in S, there's a path

in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(NTG)$; for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-resolving number," and "Setting of neutrosophic joint-resolving number," for introduced results and used classes. This approach facilitates identifying sets which form joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of joint-resolved vertices and neutrosophic cardinality of set of joint-resolved vertices corresponded to joint-resolving set have eligibility to define joint-resolving number and neutrosophic joint-resolving number but different types of set of joint-resolved vertices to define joint-resolving sets. Some results get more frameworks and more perspectives about these definitions. The way in that, different types of set of joint-resolved vertices in the terms of minimum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic joint-resolving notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Joint-Resolving Number, Neutrosophic Joint-Resolving Number,

Classes of Neutrosophic Graphs

AMS Subject Classification: 05C17, 05C22, 05E45

2.2 Background

Fuzzy set in **Ref.** [**Ref22**] by Zadeh (1965), intuitionistic fuzzy sets in **Ref.** [**Ref3**] by Atanassov (1986), a first step to a theory of the intuitionistic fuzzy graphs in **Ref.** [**Ref18**] by Shannon and Atanassov (1994), a unifying field in logics neutrosophy: neutrosophic probability, set and logic, rehoboth in **Ref.** [**Ref19**] by Smarandache (1998), single-valued neutrosophic sets in **Ref.** [**Ref21**] by Wang et al. (2010), single-valued neutrosophic graphs in **Ref.**

[**Ref5**] by Broumi et al. (2016), operations on single-valued neutrosophic graphs in **Ref.** [**Ref1**] by Akram and Shahzadi (2017), neutrosophic soft graphs in **Ref.** [**Ref17**] by Shah and Hussain (2016), bounds on the average and minimum attendance in preference-based activity scheduling in **Ref.** [**Ref2**] by Aronshtam and Ilani (2022), investigating the recoverable robust single machine scheduling problem under interval uncertainty in **Ref.** [**Ref4**] by Bold and Goerigk (2022), truncated metric dimension for finite graphs in **Ref.** [**Ref6**] by R.M. Frongillo et al. (2022), extremal graphs of bipartite graphs of given diameter for two indices on resistance-distance in **Ref.** [**Ref11**] by Y. Hong, and L. Miao (2022), a bridge between the minimal doubly resolving set problem in (folded) hypercubes and the coin weighing problem in **Ref.** [**Ref12**] by C. Lu, and Q. Ye (2022), link dimension and exact construction of graphs from distance vectors in **Ref.** [**Ref13**] by G.S. Mahindre, and A. P. Jayasumana (2022), on the robustness of the metric dimension of grid graphs to adding a single edge in **Ref.** [**Ref14**] by S. Mashkaria et al. (2022), vertex and edge metric dimensions of cacti in **Ref.** [**Ref15**] by J. Sedlar, and R. Skrekovski (2022), vertex and edge metric dimensions of unicyclic graphs in **Ref.** [**Ref16**] by J. Sedlar, and R. Skrekovski (2022), edge metric dimension and mixed metric dimension of planar graph Q_n in **Ref.** [**Ref20**] by J. Qu, and N. Cao (2022), dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [**Ref8**] by Henry Garrett (2022), three types of neutrosophic alliances based on connectedness and (strong) edges in **Ref.** [**Ref10**] by Henry Garrett (2022), properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [**Ref9**] by Henry Garrett (2022), are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [**Ref7**] by Henry Garrett (2022).
In this section, I use two subsections to illustrate a perspective about the background of this study.

2.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.3.1. *Is it possible to use mixed versions of ideas concerning "joint-resolving number", "neutrosophic joint-resolving number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having smallest number of joint-resolved vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all joint-resolved vertices in the way that, some types of numbers, joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to

neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", minimum number of joint-resolved vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, in section "Setting of joint-resolving number," as individuals. In section "Setting of joint-resolving number," joint-resolving number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of joint-resolving number," and "Setting of neutrosophic joint-resolving number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

2.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 2.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 2.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0,1]$, and $\mu_i : E \to [0,1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sum_{i=1}^{3} \sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 2.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 2.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0,1]$, and $\mu_i : E \to [0,1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 2.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1,σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1,σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1,σ_2}.

Remark 2.4.6. Using notations which is mixed with literatures, are reviewed.

2.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),\ \mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

2.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

Definition 2.4.7. (joint-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called **joint-resolving number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called **neutrosophic joint-resolving number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

82prp9

Proposition 2.4.8. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it joint-resolves.*

Proposition 2.4.9. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-resolving number if and only if for all s in S, either there are vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ or there's vertex s' in S, such that are s and s' twin vertices.*

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.4.10. In Figure (2.1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(NTG) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set

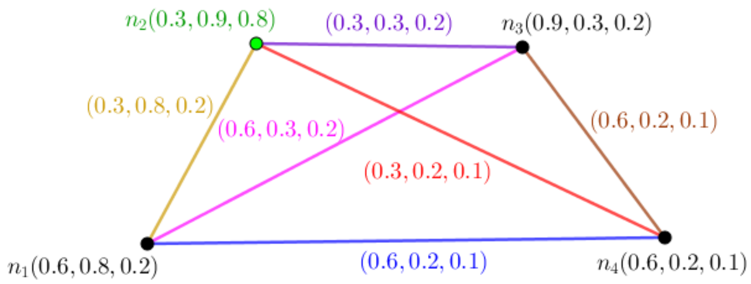

Figure 2.1: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG1

corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(NTG) = 3.9$.

2.5 Setting of joint-resolving number

In this section, I provide some results in the setting of joint-resolving number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$

For given two vertices n and n', $d(s, n) = 1 = 1 = d(s, n')$, then s doesn't joint-resolve n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$. Thus

$$\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

■

Proposition 2.5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-resolving number is equal to dominating number.*

Proposition 2.5.3. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.5.4. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$ then minus one. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma) - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.5. In Figure (2.2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.5.6. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{J}(PTH) = 1.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $n_1, n_2, \cdots, n_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, x and y, there's one path from x to y. All joint-resolving sets corresponded to

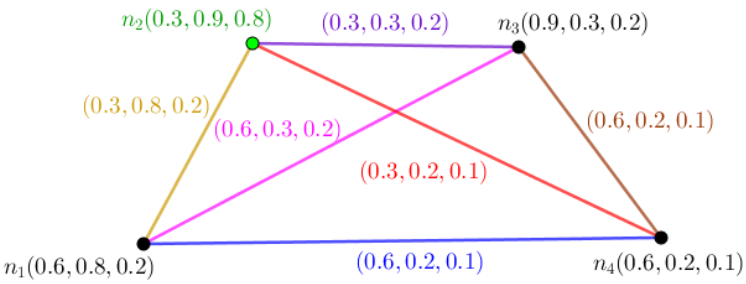

Figure 2.2: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG2

joint-resolving number are $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$. For given two vertices n_i and n_j,

$$d(n_1, n) = i \neq j = d(n_1, n_j),$$

$$d(n_{\mathcal{O}(PTH)}, n) = i \neq j = d(n_{\mathcal{O}(PTH)}, n_j),$$

then n_1 and $n_{\mathcal{O}(PTH)}$ joint-resolves n_i and n_j, where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$, is called joint-resolving set where for every two vertices in S, there's a path in S amid them, by Proposition (2.4.8), and S has one member. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(PTH) = 1.$$

Thus

$$\mathcal{J}(PTH) = 1.$$

■

Proposition 2.5.7. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are $2 \times \mathcal{O}(PTH) - 1$ joint-resolving sets.*

Proposition 2.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are two joint-resolving sets corresponded to joint-resolving number.*

Example 2.5.9. There are two sections for clarifications.

(a) In Figure (2.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given two neutrosophic vertices, s and s', there's only one path between them;

  (ii) one vertex only resolves some vertices as if not all if it isn't a leaf, then it only resolves some of all vertices and if it's a leaf, then it only resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of path, by joint-resolving set corresponded to joint-resolving number has one member and Proposition (2.4.8);

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_5\}$. For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_5\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_5 in S such that n_1 or n_5 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_5\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(PTH) = 1$;

(iv) there are nine joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$
$$\{n_1, n_2, n_3, n_4\}, \{n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_3\}, \{n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nine joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$
$$\{n_1, n_2, n_3, n_4\}, \{n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_3\}, \{n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_5\}$. For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_5\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_5 in S such that n_1 or n_5 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_5\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(PTH) = 1.2$. S is $\{n_1\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's only one path between them;

(ii) one vertex only resolves some vertices as if not all if it isn't a leaf, then it only resolves some of all vertices and if it's a leaf, then it only resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of path, by joint-resolving set corresponded to joint-resolving number has one member and Proposition (2.4.8);

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_6\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_6\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_6 in S such that n_1 or n_6 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_6\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(PTH) = 1$;

(iv) there are eleven joint-resolving sets

$$\begin{aligned}
&\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},\\
&\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6\},\\
&\{n_6, n_5\}, \{n_6, n_5, n_4\}, \{n_6, n_5, n_4, n_3\},\\
&\{n_6, n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},
\end{aligned}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are eleven joint-resolving sets

$$\begin{aligned}
&\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},\\
&\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6\},\\
&\{n_6, n_5\}, \{n_6, n_5, n_4\}, \{n_6, n_5, n_4, n_3\},\\
&\{n_6, n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},
\end{aligned}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_6\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its

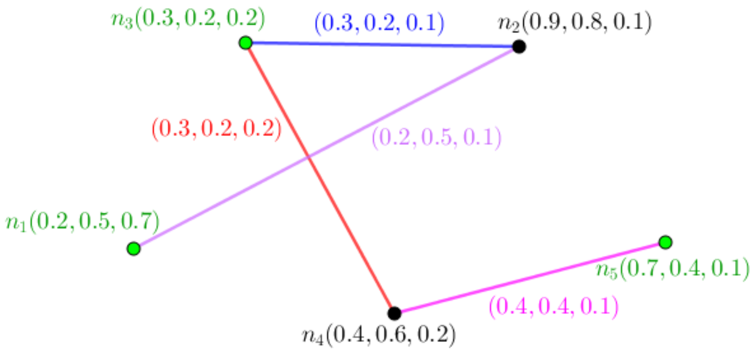

Figure 2.3: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG3

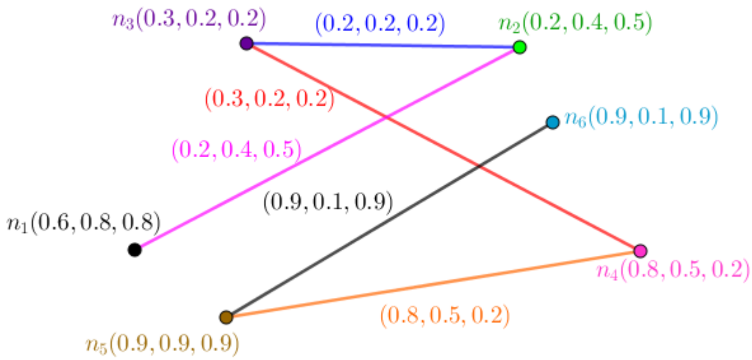

Figure 2.4: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG4

values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_6\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_6 in S such that n_1 or n_6 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_6\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(PTH) = 1.9$. S is $\{n_6\}$ corresponded to neutrosophic joint-resolving number.

Proposition 2.5.10. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}(CYC) = 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. 2 consecutive vertices could belong to S which is joint-resolving set related to joint-resolving number. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-resolving set for S. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CYC) = 2.$$

Thus

$$\mathcal{J}(CYC) = 2.$$

■

Proposition 2.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $(\mathcal{O}(CYC) \times (2^{\mathcal{O}(CYC)-2} - 1)) + 1$ joint-resolving sets.*

Proposition 2.5.12. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.13. There are two sections for clarifications.

(a) In Figure (2.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$

$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$

$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirty-six joint-resolving sets

$$\begin{aligned}
&\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},\\
&\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}\\
&\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},\\
&\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},\\
&\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},\\
&\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},\\
&\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},\\
&\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},\\
&\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},\\
&\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},\\
&\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},\\
&\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\},
\end{aligned}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\begin{aligned}
&\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},\\
&\{n_4, n_5\}, \{n_5, n_1\}.
\end{aligned}$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\begin{aligned}
&\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},\\
&\{n_4, n_5\}, \{n_5, n_1\}.
\end{aligned}$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\begin{aligned}
&\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},\\
&\{n_4, n_5\}, \{n_5, n_1\}
\end{aligned}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1, n_5\}$ corresponded to neutrosophic joint-resolving number.

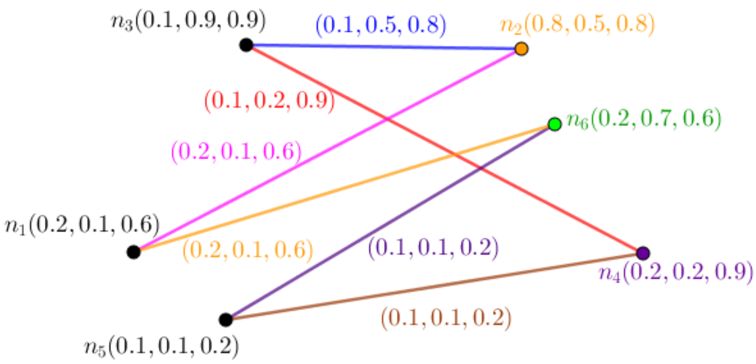

Figure 2.5: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG5

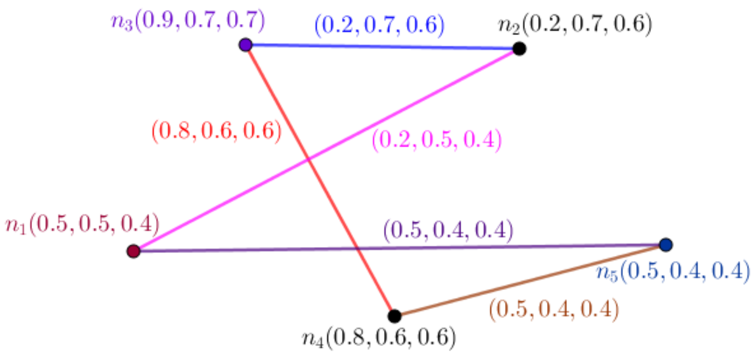

Figure 2.6: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG6

Proposition 2.5.14. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{J}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. All joint-resolving sets corresponded to joint-resolving number are

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}.$$

For every neutrosophic vertices n and n' in $V\backslash S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

Thus

$$\mathcal{J}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

■

Proposition 2.5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $\mathcal{O}(STR_{1,\sigma_2}) - 1$ joint-resolving sets.*

Proposition 2.5.16. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $\mathcal{O}(STR_{1,\sigma_2})$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.17. There is one section for clarifications. In Figure (2.7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there's only one path, precisely one edge between them and there's no path despite them;

(ii) one vertex only resolves one vertex in S, then it only resolves in S, its neighbors thus it implies the vertex joint-resolves in S, is different from a vertex resolves vertices in S, in the setting of star, by any resolving set has no center as if any joint-resolving set has to has center to hold the property from additional condition joint-resolving since if we don't have center, then there's no edge amid any given vertices in any sets;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1 = 4$;

(iv) there are five joint-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are five joint-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}$$

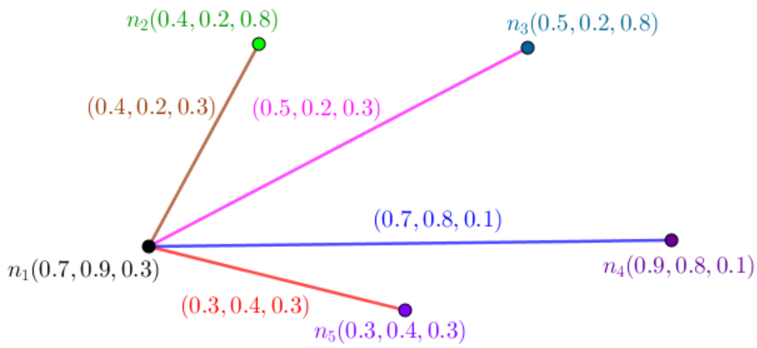

Figure 2.7: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG7

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(n_4) = 5.8.$$

S is $\{n_1, n_2, n_3, n_5\}$ corresponded to neutrosophic joint-resolving number.

Proposition 2.5.18. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph which isn't star-neutrosophic graph which means $|V_1|, |V_2| \geq 2$. Then*

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-resolved by any given vertex. Thus minimum cardinality implies excluding two vertices from different part. Consider same parity of indexes implies same part for the corresponded vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$

$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots,$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

Thus

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

■

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.19. There is one section for clarifications. In Figure (2.8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only resolves two vertices, then it only resolves its two neighbors thus it implies the vertex joint-resolves is as same as vertex resolves vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2 = 2;$$

(iv) there are nine joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\}, ,$$
$$\{n_2, n_4, n_1\}, \{n_2, n_4, n_3\}, \{n_3, n_4\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nine joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\}, ,$$
$$\{n_2, n_4, n_1\}, \{n_2, n_4, n_3\}, \{n_3, n_4\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_3)) = 2.4.$$

S is $\{n_2, n_4\}$ corresponded to neutrosophic joint-resolving number.

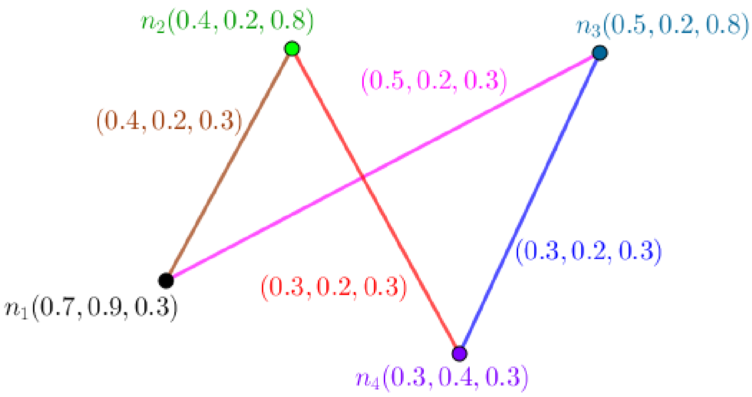

Figure 2.8: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number.

82NTG8

Proposition 2.5.20. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-t-partite-neutrosophic graph where* $t \geq 3$. *Then*

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - t.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-resolved by any given vertex. Thus minimum cardinality implies excluding t vertices from t different parts. Consider indexes implies different part for the corresponded vertices which are one, two, three, and four means they're in different parts so as the deletions of them are possible from joint-resolving sets corresponded to joint-resolving number. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots,$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - t.$$

Thus

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - t.$$

■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.21. There is one section for clarifications. In Figure (2.9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only resolves two vertices, then it only resolves its two neighbors thus it implies the vertex joint-resolves is as same as vertex resolves vertices in the setting of t-partite, by S has t members from different parts implies one edge amid them;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2 = 3;$$

(iv) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_3)) = 3.8.$$

S is $\{n_2, n_4\}$ corresponded to neutrosophic joint-resolving number.

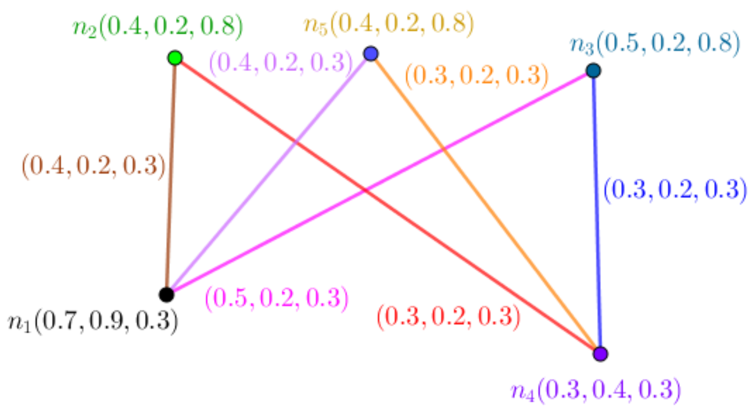

Figure 2.9: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG9

Proposition 2.5.22. *Let* $NTG : (V, E, \sigma, \mu)$ *be a wheel-neutrosophic graph. Then*

$$\mathcal{J}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 3.$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. For every vertices, the minimum number of edges amid them is either one or two because of center and the notion of neighbors. Let n_1 is the center and consecutive indexes imply consecutive vertices. Also, consider n_2 and $n_{\mathcal{O}(WHL_{1,\sigma_2})}$ are consecutive vertices without loss of generality. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\ldots$$
$$\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\ldots$$
$$\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices,

S is either of

$$\begin{aligned}
&\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},\\
&\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},\\
&\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},\\
&\ldots\\
&\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}
\end{aligned}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 3.$$

Thus

$$\mathcal{J}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 3.$$

■

Proposition 2.5.23. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $(\mathcal{O}(WHL_{1,\sigma_2}) - 3)! \times 8$ joint-resolving sets.*

Proposition 2.5.24. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $(\mathcal{O}(WHL_{1,\sigma_2}) - 3)!$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.25. There is one section for clarifications. In Figure (2.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there's only one edge between them;

(ii) one vertex resolves some vertices, as if it doesn't resolve its neighbors thus it implies the vertex joint-resolves is different from vertex resolves vertices in the setting of wheel, by S has more than one member and two vertices have two edges amid them in the cycle of wheel resolve the latter vertices out of S since minimum number of edges amid two given vertices are either one or two implying the different visions has to be applied;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\begin{aligned}
&\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},\\
&\{n_5, n_2\}.
\end{aligned}$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 3 = 2;$$

(iv) there are nineteen joint-resolving sets

$$\{n_2, n_3\}, \{n_2, n_3, n_1\}, \{n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_5\}, \{n_2, n_3, n_1, n_4\}, \{n_2, n_3, n_1, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_1, n_4, n_5\}, \{n_3, n_4\},$$
$$\{n_3, n_4, n_1\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_5\},$$
$$\{n_4, n_5\}, \{n_4, n_5, n_1\}, \{n_4, n_5, n_2\},$$
$$\{n_4, n_5, n_1, n_2\}, \{n_5, n_2\}, \{n_5, n_2, n_1\},$$
$$\{n_5, n_2, n_4\}$$

as if it's possible to have one of them

$$\{n_4, n_5\}$$

as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-resolving sets

$$\{n_2, n_3\}, \{n_2, n_3, n_1\}, \{n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_5\}, \{n_2, n_3, n_1, n_4\}, \{n_2, n_3, n_1, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_1, n_4, n_5\}, \{n_3, n_4\},$$
$$\{n_3, n_4, n_1\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_5\},$$
$$\{n_4, n_5\}, \{n_4, n_5, n_1\}, \{n_4, n_5, n_2\},$$
$$\{n_4, n_5, n_1, n_2\}, \{n_5, n_2\}, \{n_5, n_2, n_1\},$$
$$\{n_5, n_2, n_4\}$$

as if there's one joint-resolving set

$$\{n_4, n_5\}$$

corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_2) + \sigma_i(n_5))$$
$$= \sum_{i=1}^{3}(\sigma_i(n_4) + \sigma_i(n_5)) = 2.4.$$

2.6 Setting of neutrosophic joint-resolving number

In this section, I provide some results in the setting of neutrosophic joint-resolving number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected

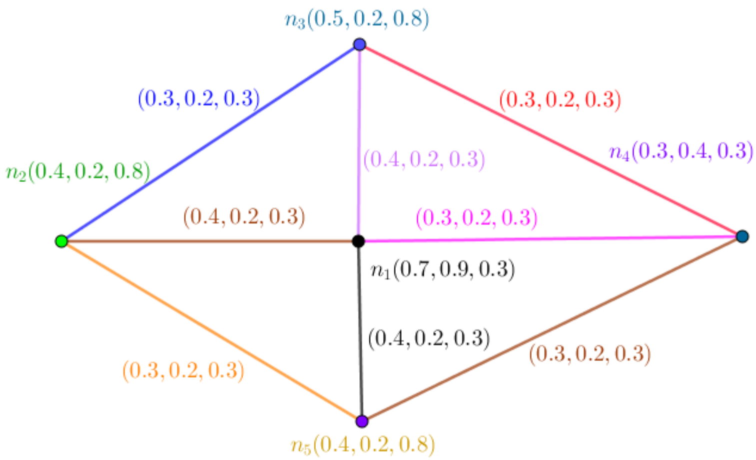

Figure 2.10: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG10

to each other. So there's one edge between two vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$

For given two vertices n and n', $d(s, n) = 1 = 1 = d(s, n')$, then s doesn't joint-resolve n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Thus

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

■

Proposition 2.6.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-resolving number is equal to dominating number.*

Proposition 2.6.3. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.6.4. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$ then minus one. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma) - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.5. In Figure (2.2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$,

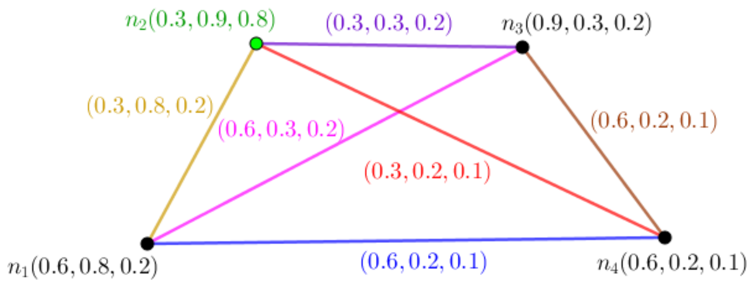

Figure 2.11: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG11

then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.6.6. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{J}_n(PTH) = \min\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \text{ is leaf.}}.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $n_1, n_2, \cdots, n_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, x and y, there's one path from x to y. All joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$. For given two vertices n_i and n_j,

$$d(n_1, n) = i \neq j = d(n_1, n_j),$$

$$d(n_{\mathcal{O}(PTH)}, n) = i \neq j = d(n_{\mathcal{O}(PTH)}, n_j),$$

then n_1 and $n_{\mathcal{O}(PTH)}$ joint-resolves n_i and n_j, where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is $\{n_1\}$ and $\{n_{\mathcal{O}(PTH)}\}$, is called joint-resolving set where for every two vertices in S, there's a path in S amid them, by Proposition (2.4.8), and S has one member.

The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(PTH) = \min\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \text{ is leaf.}}.$$

Thus

$$\mathcal{J}_n(PTH) = \min\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \text{ is leaf.}}.$$

■

Proposition 2.6.7. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are $2 \times \mathcal{O}(PTH) - 1$ joint-resolving sets.*

Proposition 2.6.8. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then there are two joint-resolving sets corresponded to joint-resolving number.*

Example 2.6.9. There are two sections for clarifications.

(a) In Figure (2.12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, s and s', there's only one path between them;
- (ii) one vertex only resolves some vertices as if not all if it isn't a leaf, then it only resolves some of all vertices and if it's a leaf, then it only resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of path, by joint-resolving set corresponded to joint-resolving number has one member and Proposition (2.4.8);
- (iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_5\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_5\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_5 in S such that n_1 or n_5 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_5\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(PTH) = 1$;
- (iv) there are nine joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$
$$\{n_1, n_2, n_3, n_4\}, \{n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_3\}, \{n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nine joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$
$$\{n_1, n_2, n_3, n_4\}, \{n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_3\}, \{n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_5\}$. For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_5\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_5 in S such that n_1 or n_5 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_5\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(PTH) = 1.2$. S is $\{n_1\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's only one path between them;

(ii) one vertex only resolves some vertices as if not all if it isn't a leaf, then it only resolves some of all vertices and if it's a leaf, then it only resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of path, by joint-resolving set corresponded to joint-resolving number has one member and Proposition (2.4.8);

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_6\}$. For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_6\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_6 in S such that n_1 or n_6 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_6\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(PTH) = 1$;

(iv) there are eleven joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$

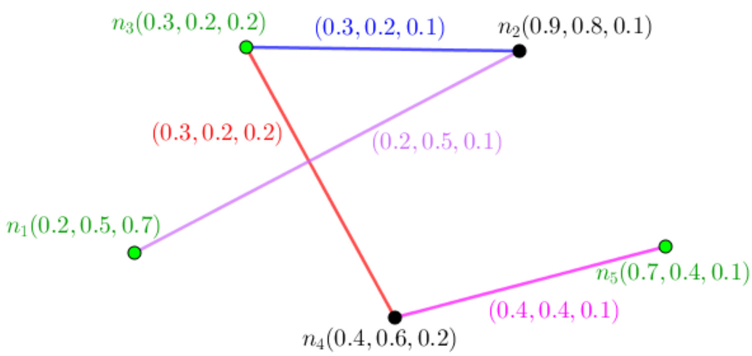

Figure 2.12: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG12

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6\},$$
$$\{n_6, n_5\}, \{n_6, n_5, n_4\}, \{n_6, n_5, n_4, n_3\},$$
$$\{n_6, n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are eleven joint-resolving sets

$$\{n_1\}, \{n_1, n_2\}, \{n_1, n_2, n_3\},$$
$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6\},$$
$$\{n_6, n_5\}, \{n_6, n_5, n_4\}, \{n_6, n_5, n_4, n_3\},$$
$$\{n_6, n_5, n_4, n_3, n_2\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are $\{n_1\}$ and $\{n_6\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1\}$ and $\{n_6\}$. For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex n_1 or n_6 in S such that n_1 or n_6 joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1\}$ and $\{n_6\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(PTH) = 1.9$. S is $\{n_6\}$ corresponded to neutrosophic joint-resolving number.

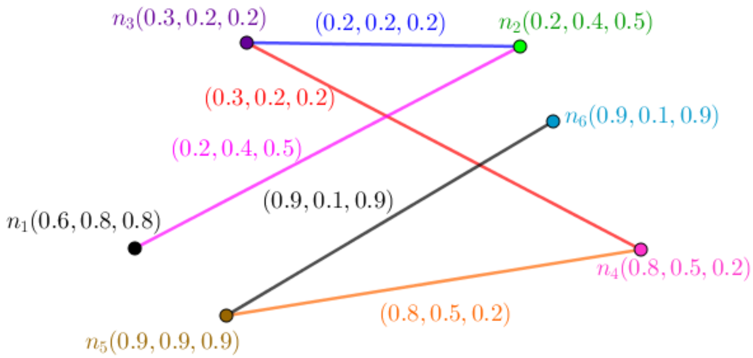

Figure 2.13: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG13

Proposition 2.6.10. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. 2 consecutive vertices could belong to S which is joint-resolving set related to joint-resolving number. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-resolving set for S. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

Thus

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

■

Proposition 2.6.11. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $(\mathcal{O}(CYC) \times (2^{\mathcal{O}(CYC)-2} - 1)) + 1$ joint-resolving sets.*

Proposition 2.6.12. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.13. There are two sections for clarifications.

(a) In Figure (2.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, there are only two paths between them;
- (ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;
- (iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$

$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$

$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$

$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1, n_5\}$ corresponded to neutrosophic joint-resolving number.

Proposition 2.6.14. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V \text{ and } x \text{ isn't center.}}.$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. All joint-resolving sets corresponded to joint-resolving number are

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and

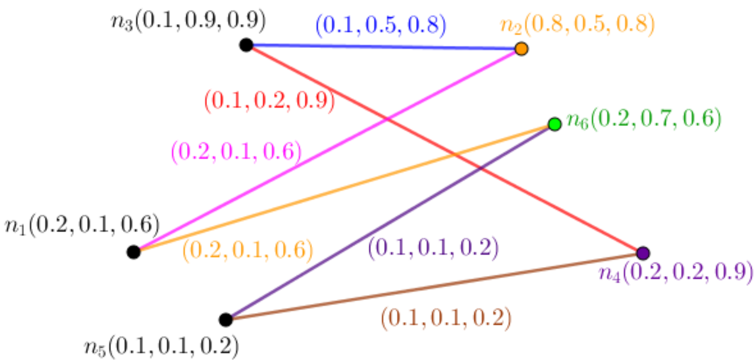

Figure 2.14: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG14

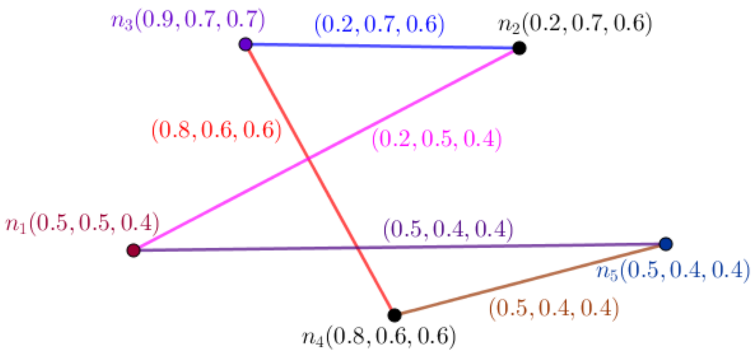

Figure 2.15: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG15

the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}, \{c, n_2, n_3, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$
$$\{c, n_3, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\}, \ldots, \{c, n_2, n_4, \ldots, n_{\mathcal{O}(STR_{1,\sigma_2})}\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V \text{ and } x \text{ isn't center.}}.$$

Thus

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V \text{ and } x \text{ isn't center.}}.$$

■

Proposition 2.6.15. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $\mathcal{O}(STR_{1,\sigma_2}) - 1$ joint-resolving sets.*

Proposition 2.6.16. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then there are $\mathcal{O}(STR_{1,\sigma_2})$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.17. There is one section for clarifications. In Figure (2.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there's only one path, precisely one edge between them and there's no path despite them;

(ii) one vertex only resolves one vertex in S, then it only resolves in S, its neighbors thus it implies the vertex joint-resolves in S, is different from a vertex resolves vertices in S, in the setting of star, by any resolving set has no center as if any joint-resolving set has to has center to hold the property from additional condition joint-resolving since if we don't have center, then there's no edge amid any given vertices in any sets;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$

$$\{n_1, n_2, n_4, n_5\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1 = 4$;

(iv) there are five joint-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are five joint-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_5\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(n_4) = 5.8.$$

S is $\{n_1, n_2, n_3, n_5\}$ corresponded to neutrosophic joint-resolving number.

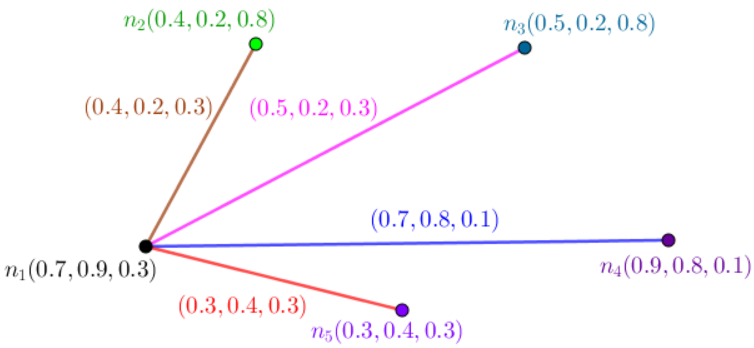

Figure 2.16: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG16

Proposition 2.6.18. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph which isn't star-neutrosophic graph which means $|V_1|, |V_2| \geq 2$. Then*

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \max\{\sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y))\}_{x \text{ and } y \text{ are in different parts}}.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-resolved by any given vertex. Thus minimum cardinality implies excluding two vertices from different part. Consider same parity of indexes implies same part for the corresponded vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_3, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_5, x_6, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots,$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \max\{\sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y))\}_{x \text{ and } y \text{ are in different parts}}.$$

Thus

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \max\{\sum_{i=1}^{3}(\sigma_i(x)+\sigma_i(y))\}_{x \text{ and } y \text{ are in different parts.}}.$$

■

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.19. There is one section for clarifications. In Figure (2.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only resolves two vertices, then it only resolves its two neighbors thus it implies the vertex joint-resolves is as same as vertex resolves vertices in the setting of bipartite, by S has two members from different parts implies one edge amid them;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2 = 2;$$

(iv) there are nine joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\}, ,$$
$$\{n_2, n_4, n_1\}, \{n_2, n_4, n_3\}, \{n_3, n_4\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nine joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_3\}, \{n_1, n_3, n_4\}, \{n_2, n_4\}, ,$$
$$\{n_2, n_4, n_1\}, \{n_2, n_4, n_3\}, \{n_3, n_4\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_2, n_4\},$$
$$\{n_3, n_4\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_3)) = 2.4.$$

S is $\{n_2, n_4\}$ corresponded to neutrosophic joint-resolving number.

Proposition 2.6.20. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \ldots + \sigma_i(x_t))\}_{x_1,\ldots,x_t \text{ are in different parts.}}.$$

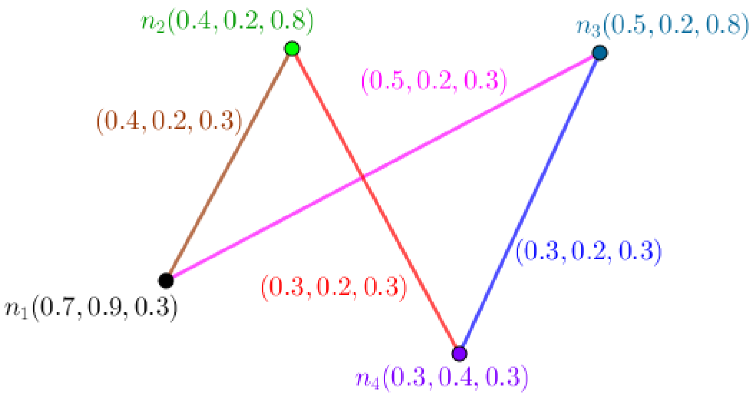

Figure 2.17: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG17

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is joint-resolved by any given vertex. Thus minimum cardinality implies excluding t vertices from t different parts. Consider indexes implies different part for the corresponded vertices which are one, two, three, and four means they're in different parts so as the deletions of them are possible from joint-resolving sets corresponded to joint-resolving number. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_1, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\},$$
$$\{x_2, x_t, x_{t+1}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \{x_3, x_4, x_{t+3}, x_{t+2}, \ldots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}\}, \ldots,$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \ldots + \sigma_i(x_t))\}_{x_1,\ldots,x_t \text{ are in different parts.}}.$$

Thus

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \ldots + \sigma_i(x_t))\}_{x_1,\ldots,x_t \text{ are in different parts.}}.$$

■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.21. There is one section for clarifications. In Figure (2.18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only resolves two vertices, then it only resolves its two neighbors thus it implies the vertex joint-resolves is as same as vertex resolves vertices in the setting of t-partite, by S has t members from different parts implies one edge amid them;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2 = 3;$$

(iv) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_3)) = 3.8.$$

S is $\{n_2, n_4\}$ corresponded to neutrosophic joint-resolving number.

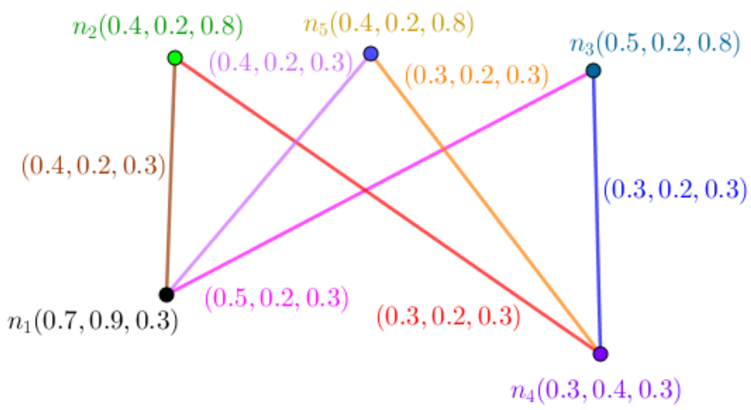

Figure 2.18: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG18

Proposition 2.6.22. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(c) + \sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices and } c \text{ is center.}}.$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. For every vertices, the minimum number of edges amid them is either one or two because of center and the notion of neighbors. Let n_1 is the center and consecutive indexes imply consecutive vertices. Also, consider n_2 and $n_{\mathcal{O}(WHL_{1,\sigma_2})}$ are consecutive vertices without loss of generality. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\ldots$$
$$\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\ldots$$
$$\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_2, n_3, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_3, n_4, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\{n_4, n_5, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3},$$
$$\ldots$$
$$\{n_{\mathcal{O}(WHL_{1,\sigma_2})}, n_2, \ldots, n_{t-1}, n_t\}_{|S|=\mathcal{O}(WHL_{1,\sigma_2})-3}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(c) + \sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices and } c \text{ is center.}}.$$

Thus

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) -$$
$$\max\{\sum_{i=1}^{3}(\sigma_i(c) + \sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices and } c \text{ is center.}}.$$

■

Proposition 2.6.23. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $(\mathcal{O}(WHL_{1,\sigma_2}) - 3)! \times 8$ joint-resolving sets.*

Proposition 2.6.24. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then there are $(\mathcal{O}(WHL_{1,\sigma_2}) - 3)!$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.25. There is one section for clarifications. In Figure (2.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and n_1, there's only one edge between them;

(ii) one vertex resolves some vertices, as if it doesn't resolve its neighbors thus it implies the vertex joint-resolves is different from vertex resolves vertices in the setting of wheel, by S has more than one member and two vertices have two edges amid them in the cycle of wheel resolve the latter vertices out of S since minimum number of edges amid two given vertices are either one or two implying the different visions has to be applied;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 3 = 2;$$

(iv) there are nineteen joint-resolving sets

$$\{n_2, n_3\}, \{n_2, n_3, n_1\}, \{n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_5\}, \{n_2, n_3, n_1, n_4\}, \{n_2, n_3, n_1, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_1, n_4, n_5\}, \{n_3, n_4\},$$
$$\{n_3, n_4, n_1\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_5\},$$
$$\{n_4, n_5\}, \{n_4, n_5, n_1\}, \{n_4, n_5, n_2\},$$
$$\{n_4, n_5, n_1, n_2\}, \{n_5, n_2\}, \{n_5, n_2, n_1\},$$
$$\{n_5, n_2, n_4\}$$

as if it's possible to have one of them

$$\{n_4, n_5\}$$

as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-resolving sets

$$\{n_2, n_3\}, \{n_2, n_3, n_1\}, \{n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_5\}, \{n_2, n_3, n_1, n_4\}, \{n_2, n_3, n_1, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_2, n_3, n_1, n_4, n_5\}, \{n_3, n_4\},$$
$$\{n_3, n_4, n_1\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_5\},$$

$$\{n_4, n_5\}, \{n_4, n_5, n_1\}, \{n_4, n_5, n_2\},$$
$$\{n_4, n_5, n_1, n_2\}, \{n_5, n_2\}, \{n_5, n_2, n_1\},$$
$$\{n_5, n_2, n_4\}$$

as if there's one joint-resolving set

$$\{n_4, n_5\}$$

corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_2, n_3\}, \{n_3, n_4\}, \{n_4, n_5\},$$
$$\{n_5, n_2\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3}(\sigma_i(n_1) + \sigma_i(n_2) + \sigma_i(n_5))$$
$$= \sum_{i=1}^{3}(\sigma_i(n_4) + \sigma_i(n_5)) = 2.4.$$

2.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common

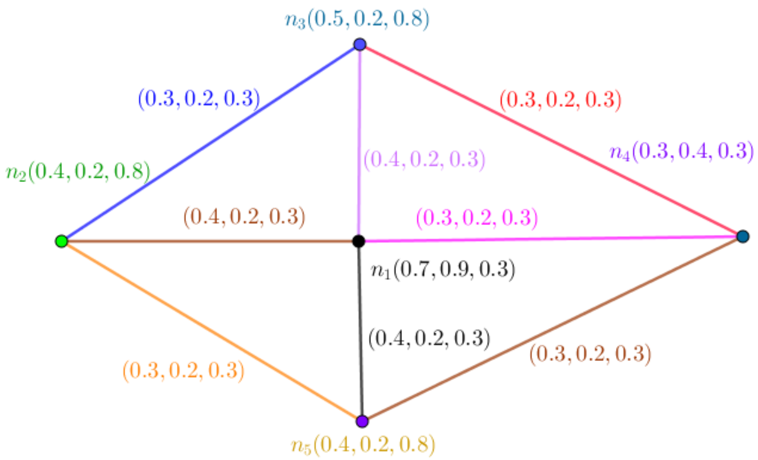

Figure 2.19: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG19

neutrosophic vertex set.
Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 2.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 82tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

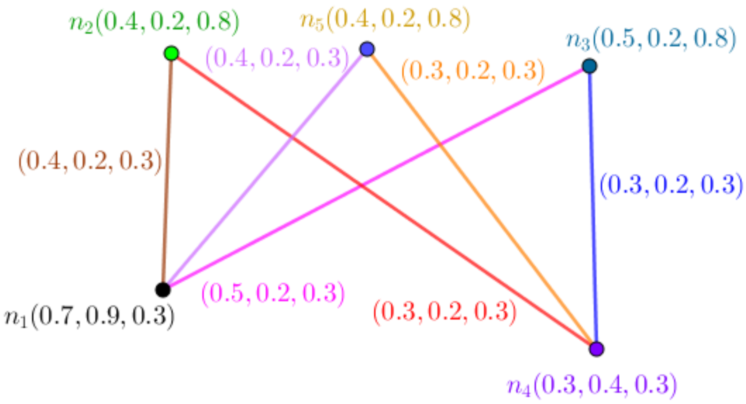

Figure 2.20: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number

82NTG20

2.8 Case 1: Complete-t-partite Model alongside its joint-resolving number and its neutrosophic joint-resolving number

Step 4. (Solution) The neutrosophic graph alongside its joint-resolving number and its neutrosophic joint-resolving number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its joint-resolving number and its neutrosophic joint-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (2.20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its joint-resolving number and its neutrosophic joint-resolving number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.20). In Figure (2.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, n and n', there is either one path with length one or one path with length two between them;

(ii) one vertex only resolves two vertices, then it only resolves its two neighbors thus it implies the vertex joint-resolves is as same as vertex resolves vertices in the setting of t-partite, by S has t members from different parts implies one edge amid them;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\},$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2 = 3;$$

(iv) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirteen joint-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_5\}, \{n_1, n_3, n_5, n_4\}, \{n_4, n_2, n_3\},$$
$$\{n_4, n_2, n_3, n_5\}, \{n_4, n_2, n_5\}, \{n_4, n_2, n_5, n_1\},$$
$$\{n_4, n_3, n_5\}$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_5\}, \{n_1, n_3, n_5\},$$
$$\{n_4, n_2, n_3\}, \{n_4, n_2, n_5\}, \{n_4, n_3, n_5\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \sum_{i=1}^{3}(\sigma_i(n_1)+\sigma_i(n_3)) = 3.8.$$

S is $\{n_2, n_4\}$ corresponded to neutrosophic joint-resolving number.

2.9 Case 2: Complete Model alongside its Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number

Step 4. (Solution) The neutrosophic graph alongside its joint-resolving number and its neutrosophic joint-resolving number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its joint-resolving number and its neutrosophic joint-resolving number when the notion of family is applied in the way that

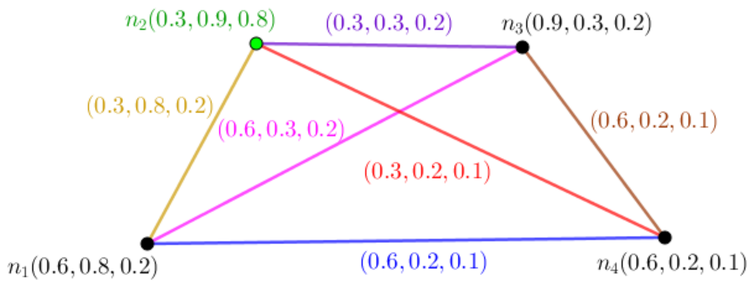

Figure 2.21: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number

82NTG21

all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (2.21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its joint-resolving number and its neutrosophic joint-resolving number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.21). There is one section for clarifications.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

2.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its joint-resolving number and its neutrosophic joint-resolving number are defined in neutrosophic graphs. Thus,

Question 2.10.1. *Is it possible to use other types of its joint-resolving number and its neutrosophic joint-resolving number?*

Question 2.10.2. *Are existed some connections amid different types of its joint-resolving number and its neutrosophic joint-resolving number in neutrosophic graphs?*

Question 2.10.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 2.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 2.10.5. *Which parameters are related to this parameter?*

Problem 2.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

2.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses two definitions concerning joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of joint-resolved vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of joint-resolved vertices corresponded to joint-resolving set is called neutrosophic joint-resolving number. The connections of vertices which aren't clarified by minimum number of edges amid them differ them from each other and put them in different categories to represent a number which is

Table 2.2: A Brief Overview about Advantages and Limitations of this Study 82tbl

Advantages	Limitations
1. Joint-Resolving Number of Model	1. Connections amid Classes
2. Neutrosophic Joint-Resolving Number of Model	
3. Minimal Joint-Resolving Sets	2. Study on Families
4. Joint-Resolved Vertices amid all Vertices	
5. Acting on All Vertices	3. Same Models in Family

called joint-resolving number and neutrosophic joint-resolving number arising from joint-resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

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